

this is a greedy algorithm for a discrete optimization problem, there is no guarantee that the global maximum will be reached. However, the theorem demonstrates that the effective independence distribution is an exact measure of the information that will be lost when deleting one potential sensor location, and in practical applications the method has been shown<sup>1,2</sup> to be effective for optimally locating sensors.

### References

- <sup>1</sup>Kammer, D. C., "Sensor Placement for On-Orbit Modal Identification and Correlation of Large Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 2, 1991, pp. 251-259.
- <sup>2</sup>Poston, W. L., "Optimal Sensor Locations for On-Orbit Modal Identification of Large Space Structures," M.S. Thesis, Dept. of Civil, Mechanical, and Environmental Engineering, George Washington Univ., Hampton, VA, July 1991.
- <sup>3</sup>Strang, G., *Linear Algebra and Its Applications*, 3rd ed., Harcourt Brace Jovanovich, San Diego, CA, 1988, pp. 214-234.

## Reply by Author to W. L. Poston and R. H. Tolson

Daniel C. Kammer\*

University of Wisconsin, Madison, Wisconsin 53706

THE author would like to thank Poston and Tolson for their interest in the effective independence method of sensor placement he presented in Ref. 1. It was believed that the effective independence method ranked candidate sensor locations such that the deletion of the lowest ranked sensor resulted in the smallest change in the determinant of the Fisher information matrix (FIM). However, this hypothesis had not been proven prior to Poston and Tolson's comment. Their result is elegant and its proof convincing; however, the author would like to present an alternate proof recently brought to his attention by L. Yao, a graduate student in the Department of Electrical and Computer Engineering at the University of Wisconsin. The proof starts with a lemma from Ref. 2.

**Lemma:** Let  $C \in R^{n \times m}$ ,  $D \in R^{m \times n}$ , and  $I_p$  be a  $p \times p$  identity matrix. Then

$$\det(I_n - CD) = \det(I_m - DC) \quad (1)$$

**Proof:** See the Appendix in Ref. 2.  $\square$

The following theorem states Poston and Tolson's result in a slightly different way:

**Theorem:**  $\forall A = \Phi^T \Phi \in R^{n \times n}$  and  $A$  positive definite. Let  $r_i \in R^{1 \times n}$  be the  $i$ th row vector of  $\Phi$  and  $B = A - r_i^T r_i$ . Then

$$\det(B) = \det(A)(1 - E_{Di}) \quad (2)$$

where  $0 \leq E_{Di} \leq 1$ .

Note that  $r_i = R_i^T$ , which is used in both the comment and Ref. 1.

**Proof:**

$$\begin{aligned} \det(B) &= \det(A - r_i^T r_i) \\ &= \det[A(I - A^{-1} r_i^T r_i)] \end{aligned} \quad (3)$$

Since  $A$  and  $(I - A^{-1} r_i^T r_i)$  are both square matrices,

$$\begin{aligned} \det(B) &= \det(A) \det(I - A^{-1} r_i^T r_i) \\ &= \det(A) \det(1 - r_i A^{-1} r_i^T) = \det(A)(1 - E_{Di}) \end{aligned} \quad (4)$$

where the foregoing lemma has been used and  $E_{Di} = r_i A^{-1} r_i^T$ . Recall that

$$B = A - r_i^T r_i = \sum_{\substack{j=1 \\ j \neq i}}^n r_j^T r_j = \Gamma_i^T \Gamma_i \quad (5)$$

where  $\Gamma_i = [r_1^T, \dots, r_{i-1}^T, r_{i+1}^T, \dots, r_n^T]^T$ . Since matrix  $B$  can be expressed in this factored form, it must be positive semidefinite. This implies that  $\det(B) \geq 0$  and thus  $E_{Di} \leq 1$ . Because  $A$  is assumed to be positive definite,  $A^{-1}$  is also positive definite. Therefore,  $\forall r_i \neq 0, i = 1, \dots, n$ ,

$$E_{Di} = r_i A^{-1} r_i^T > 0 \quad (6)$$

However,  $r_i$  could be a zero row in  $\Phi$ ; therefore,  $E_{Di} \geq 0$ . This completes the proof.  $\square$

Thus, the effective independence sensor placement method iteratively deletes candidate sensor locations that have the smallest impact on the value of the Fisher information matrix determinant.

### References

- <sup>1</sup>Kammer, D. C., "Sensor Placement for On-Orbit Modal Identification and Correlation of Large Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 2, 1991, pp. 251-259.
- <sup>2</sup>Kailath, T., *Linear Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1980.

## Errata

### Minimizing Selective Availability Error on Satellite and Ground Global Positioning System Measurements

S. C. Wu, W. I. Bertiger, and J. T. Wu  
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109

[J. Guidance 15(5), pp. 1306-1309 (1992)]

**B**EGINNING with the first full paragraph on page 1308 and continuing on page 1309, six paragraphs appear out of order. The AIAA Editorial Staff regrets this error and any inconvenience it has caused our readers. The correct order appears below:

Since carrier-phase data noise is intrinsically low, smoothing over the entire 5-min integration time is not necessary. Instead of removing the satellite dynamics with a good model, a low-order polynomial interpolation over a short time period can be used for the compression of carrier phase. The simulation analysis in the following section indicates that for Topex data at 1-s intervals, a cubic interpolation over four points every 5 min is appropriate even with the strong Topex dynamics. For ground data, a compression scheme with a quadratic